

The Family of Objective Functions in DML

CVPR Tutorial: Deep Visual Similarity and Metric Learning

Timo Milbich





Technical University of Munich



Visual Similarity Learning

Learn **representation** $\phi(x)$ which reflects **semantic similarity** $d(\phi(x_i), \phi(x_j))$ within training distribution $\mathcal{X}_{\text{train}}$.





Encoder fs.a. ResNet50, Inception-BN



Input space (Images)



Embedding space $\Phi = \mathbb{S}^D = \{z \in \mathbb{R}^D \colon \|z\|_2^2 = 1\} \;.$

Deep Metric Learning (DML)

Learn **representation** $\phi(x)$ which reflects **semantic similarity** $d(\phi(x_i), \phi(x_j))$ within training distribution $\mathcal{X}_{\text{train}}$.



Input space (Images)

Ranking-based DML

Classification-based DML

Proxy-based DML

Contextual DML

Ensemble-based DML

Others

Deep Metric Learning (DML)

Learn **representation** $\phi(x)$ which reflects **semantic similarity** $d(\phi(x_i), \phi(x_j))$ within training distribution $\mathcal{X}_{\text{train}}$.



Input space (Images)

Ranking-based DML

Classification-based DML

Proxy-based DML

Contextual DML

Ensemble-based DML

Others

• Learning informative embedding space requires to "arrange" data under $\phi(x)$



Learning informative embedding space requires to "arrange" data under $\phi(x)$



- Learning informative embedding space requires to "arrange" data under $\phi(x)$
- Naturally formulate the learning problem as orderings "A is closer to B then C"





- Learning informative embedding space requires to "arrange" data under $\phi(x)$
- Naturally formulate the learning problem as orderings "A is closer to B then C"







- Learning informative embedding space requires to "arrange" data under $\phi(x)$
- Naturally formulate the learning problem as orderings "A is closer to B then C"

Triplet loss
1
: $t=\{x_a,x_p,x_n\}$ $\ell^{ ext{triplet}}(a,p,n)=||\phi(x_a)-\phi(x_p)||_2^2+$



¹ Weinberger et al. 2006; Schroff et al. 2015

$-\left|\left|\phi(x_a)-\phi(x_n) ight| ight|_2^2$

- Learning informative embedding space requires to "arrange" data under $\phi(x)$
- Naturally formulate the learning problem as orderings "A is closer to B then C"

Triplet loss
1
: $t = \{x_a, x_p, x_n\}$ $\ell^{ ext{triplet}}(a, p, n) = \max(||\phi(x_a) - \phi(x_p)|)$



¹ Weinberger et al. 2006; Schroff et al. 2015

$|\psi_p)||_2^2 - ||\phi(x_a) - \phi(x_n)||_2^2 + lpha, 0)|_2$

fixed margin

- Learning informative embedding space requires to "arrange" data under $\phi(x)$
- Naturally formulate the learning problem as orderings "A is closer to B then C"



fixed margin



- Learning informative embedding space requires to "arrange" data under $\phi(x)$
- Naturally formulate the learning problem as orderings "A is closer to B then C"

$$\begin{array}{l} \text{minimize} \sum_{(i,j)} \ell^{\text{margin}}(i,j) + \nu \left(\beta^{(0)} - \ell^{\text{margin}}(i,j) \right) = (\alpha + y_{ij}) \\ \\ \ell^{\text{margin}}(i,j) := (\alpha + y_{ij}) \\ \\ \beta(i) := \beta^{(0)} + \beta^{(\text{class})}_{c(i)} + \ell^{(\text{class})}_{c(i)} + \ell^{(\text{class$$



- Learning informative embedding space requires to "arrange" data under $\phi(x)$
- Naturally formulate the learning problem as orderings "A is closer to B then C"

minimize
$$\sum_{(i,j)} \ell^{\text{margin}}(i,j) + \nu \left(\beta^{(0)} - \ell^{\text{margin}}(i,j) := (\alpha + y_{ij}) \left(\beta^{(i)}(i) := \beta^{(0)} + \beta^{(\text{class})}_{c(i)} + \beta^{(1)}(i)\right)$$



• So far: Per sample in batch: form a single triplet





(b) Triplet embedding

Images partly taken from: Song et al. 2015

- So far: Per sample in batch: form a single triplet
- Use all possible relations within batch for learning!





Images partly taken from: Song et al. 2015

$\mathcal{O}(B^2)$ relations

$\mathcal{O}(B)$ relations

- So far: Per sample in batch: **form a single triplet**
- Use **all possible relations** within batch for learning!



Lift

ted Structures Feature Embedding Loss³:

$$\tilde{J}_{i,j} = \log\left(\sum_{(i,k)\in\mathcal{N}} \exp\{\alpha - D_{i,k}\} + \sum_{(j,l)\in\mathcal{N}} \exp\{\alpha - D_{j,l}\}\right) + D_{i,j}$$

$$\tilde{J} = \frac{1}{2|\mathcal{P}|} \sum_{(i,j)\in\mathcal{P}} \max\left(0, \ \tilde{J}_{i,j}\right)^2,$$



(c) Lifted structured embedding

³ Song et al. 2015

- So far: Per sample in batch: **form a single triplet**
- Use **all possible relations** within batch for learning!







(c) Lifted structured embedding

³ Song et al. 2015

- So far: Per sample in batch: **form a single triplet**
- Use **all possible relations** within batch for learning!



Multi-Similarity Loss⁴:





(c) Lifted structured embedding

⁴ Wang et al. 2019



 $\mathcal{O}(B^2)$ relations

- So far: Per sample in batch: **form a single triplet**
- Use **all possible relations** within batch for learning!



Multi-Similarity Loss⁴:



Informative pair mining:

$$S_{ij}^+ < \max_{y_k \neq y_i} S_{ik} + \epsilon$$
. "positive less similar to
 $S_{ij}^- > \min_{y_k = y_i} S_{ik} - \epsilon$, "negative more similar





(b) Triplet embedding



(c) Lifted structured embedding

anchor than best negative"

to anchor than worst positive"

 \mathbf{x}_6



 $\mathcal{O}(B^2)$ relations

- Infeasible amount of triplets to learn from (Scale $\mathcal{O}(n^3)$)



- Infeasible amount of triplets to learn from (Scale $\mathcal{O}(n^3)$)
- Fixed sampling heuristics:

- **(Semi-)Hard negative mining** [Schroff et al. 2015] $n^* = \arg\min_{n:d_{\mathrm{an}}>d_{\mathrm{ap}}} d_{\mathrm{an}}$





- Infeasible amount of triplets to learn from (Scale $\mathcal{O}(n^3)$)
- Fixed sampling heuristics:
 - (Semi-)Hard negative mining [Schroff et al. 2015]

 $n^* = rg\min_{n:d_{ ext{an}} > d_{ ext{ap}}} d_{ ext{an}}$

• May suffer from **unstable gradients** due to focus on hardest negatives only





- Infeasible amount of triplets to learn from (Scale $\mathcal{O}(n^3)$)
- Fixed sampling heuristics:

- Distance-weighted sampling [Wu et al. 2017]

$$n^* \sim p(I_n | I_a) = p(d_{\mathrm{an}})$$



- Infeasible amount of triplets to learn from (Scale $\mathcal{O}(n^3)$)
- Fixed sampling heuristics:
 - Distance-weighted sampling [Wu et al. 2017] $n^* \sim p(I_n | I_a) = p(d_{ ext{an}})$
 - Soften hard negative constraint by uniform sampling from entire range of distances d_{an}



- Infeasible amount of triplets to learn from (Scale $\mathcal{O}(n^3)$)
- Fixed sampling heuristics:
 - Distance-weighted sampling [Wu et al. 2017] $n^* \sim p(I_n | I_a) = p(d_{ ext{an}})$
 - Soften hard negative constraint by uniform sampling from entire range of distances d_{an}





- Infeasible amount of triplets to learn from (Scale $\mathcal{O}(n^3)$)
- Fixed sampling heuristics:
 - Distance-weighted sampling [Wu et al. 2017] $n^* \sim p(I_n | I_a) \propto \min(\lambda, q(d_{an})^{-1})$
 - Soften hard negative constraint by uniform sampling from entire range of distances d_{an}
 - Uniform distribution on \mathbb{S}^D : $q(d) \propto d^{k-2} [1 rac{1}{4} d^2]^{rac{k-3}{2}}$





- Infeasible amount of triplets to learn from (Scale $\mathcal{O}(n^3)$)
- Fixed sampling heuristics:
 - Distance-weighted sampling [Wu et al. 2017] $n^* \sim p(I_n | I_a) \propto \min(\lambda, q(d_{ ext{an}})^{-1})$
 - Soften hard negative constraint by uniform sampling from entire range of distances d_{an}
 - Uniform distribution on \mathbb{S}^D : $q(d) \propto d^{k-2} [1 rac{1}{4}d^2]^{rac{k-3}{2}}$





- Infeasible amount of triplets to learn from (Scale $\mathcal{O}(n^3)$)
- Fixed sampling heuristics:

- (Semi-)Hard negative mining [Schroff et al. 2015]

- Distance-weighted sampling [Wu et al. 2017]

- Drawbacks:
 - Predefined and independent of DML optimization
 - Fixed and disconnected from learning process

(see Curriculum Learning)

- Optimal distribution $p(d_{\mathrm{an}})$?







• **Classification** is "classical" approach to representation learning



Images partly taken from: Zhai, Wu et al. 2019

 $\sigma(\phi(x_i)) = -\log\left[rac{\exp(W_{y_i}^ op \phi(x_i))}{\sum_{k=1}^K \exp(W_k^ op \phi(x_i))}
ight]$



Softmax Loss :

$$\sigma(\phi(x_i)) = -\log \left[rac{1}{\sum_i}
ight]$$

Classification is "classical" approach to representation learning Rows of weights matrix W_k act as class distribution proxies

 \implies alleviate the sampling problem of ranking-based methods

(Connections to **Proxy-based DML**)

 $\exp(W_{y_i}^ op \phi(x_i))$



⁶Zhai, Wu et al. 2019

Classification is "classical" approach to representation learning Rows of weights matrix W_k act as class distribution proxies

 \implies alleviate the sampling problem of ranking-based methods

(Connections to **Proxy-based DML**)

Classification as **competitive baseline**⁶: **Regularize** embedding space to





Classification is "classical" approach to representation learning Rows of weights matrix W_k act as class distribution proxies

 \implies alleviate the sampling problem of ranking-based methods

- hyphersphere

Normalized Softmax Loss :

$$\sigma(\phi(x_i)) = -\log{\left[rac{1}{\sum_k^R}
ight.}$$

(Connections to **Proxy-based DML**)

Classification as **competitive baseline**⁶: **Regularize** embedding space to

Temperature scaling to enforce

compact intra-class clusters


- So far, we optimize actual distances (e.g. Euclidean, Cosine) between data samples/proxies
- Now: Express learning constraints explicitly as actual angles on (Hyphersphere-)Manifold
- very popular for Face recognition applications



$$\sigma(\phi(x_i)) = -\log \left[rac{\exp(W_{y_i}^ op \phi(x_i))}{\sum_{k=1}^K \exp(W_k^ op \phi(x_i))}
ight]$$

Images partly taken from: Liu et al. 2018



- So far, we optimize actual **distances** (e.g. Euclidean, Cosine) between data samples/proxies
- Now: Express learning constraints explicitly as actual angles on (Hyphersphere-)Manifold
- very popular for Face recognition applications





- So far, we optimize actual distances (e.g. Euclidean, Cosine) between data samples/proxies
- Now: Express learning constraints explicitly as actual angles on (Hyphersphere-)Manifold
- very popular for Face recognition applications



$$egin{aligned} &\sigma(\phi(x_i)) = -\log\left[rac{\exp(W_{y_i}^ op\phi(x_i))}{\sum_{k=1}^K \exp(W_k^ op)}
ight] \ &= -\log\left[rac{\exp(\|W_{y_i}\|\,\|}{\sum_{k=1}^K \exp(\|W_k\|)}
ight] \end{aligned}$$

• Constrain
$$\|W_{y_i}\| = 1$$
 and $\|\phi(x_i)\|_2 = s$
 $\sigma(\phi(x_i)) = -\log\left[rac{\exp(s\cos(heta_{i,y_i}))}{\sum_{k=1}^K \exp(s\cos(heta_{i,k}))}
ight].$

We now optimize the angles $heta_{i,k}$ between $\phi(x_i)$ and W_k

Images partly taken from: Liu et al. 2018



- So far, we optimize actual distances (e.g. Euclidean, Cosine) between data samples/proxies
- Now: Express learning constraints explicitly as actual angles on (Hyphersphere-)Manifold
- very popular for Face recognition applications



- Euclidean Margin Loss
- Modified Softmax Loss
- A-Softmax Loss ($m \ge 2$)

$$egin{aligned} \sigma(\phi(x_i)) &= -\log\left[rac{\exp(W_{y_i}^ op \phi(x_i))}{\sum_{k=1}^K \exp(W_k^ op \phi(x_i))}
ight] \ &= -\log\left[rac{\exp(\|W_{y_i}\|\,\|\phi_k))}{\sum_{k=1}^K \exp(\|W_k)}
ight] \end{aligned}$$

• Constrain
$$\|W_{y_i}\| = 1$$
 and $\|\phi(x_i)\|_2 = s$ $\sigma(\phi(x_i)) = -\log\left[rac{\exp(s\cos(heta_{i,y_i}))}{\sum_{k=1}^K \exp(s\cos(heta_{i,k}))}
ight].$

- We now optimize the angles $heta_{i,k}$ between $\phi(x_i)$ and W_k
- Introduce a margin β similar to Ranking-based DML

$$\begin{array}{l} \textbf{Sphere}^{7}\text{-/ArcFace}^{8} \text{ Loss:} \\ \sigma(\phi(x_{i});\beta) = -\log \left[\frac{\exp(s\cos(\beta + \theta_{i,y_{i}}))}{\exp(s\cos(\beta + \theta_{i,y_{i}})) + \sum_{k=1, k \neq y_{i}}^{M} \exp(s\cos(\theta_{i,k}))} \right] \end{array}$$



- So far, we optimize actual distances (e.g. Euclidean, Cosine) between data samples/proxies
- Now: Express learning constraints explicitly as actual angles on (Hyphersphere-)Manifold
- very popular for Face recognition applications

Many different extensions based on this formulations!

Sphere⁷-/ArcFace⁸ Loss:
$$\sigma(\phi(x_i); eta) = -\log\left[rac{1}{\exp(s\cos(eta+s))}
ight]$$



⁷Liu et al. 2018; ⁸Deng, Guo et al. 2019

 $\exp(s\cos(eta+ heta_{i,y_i}))$ $(\overline{\theta_{i,y_i}})) + \sum_{k=1,k
eq y_i}^M \exp(s\cos(heta_{i,k}))$

• Learner ensembles are common approach to improve overall performance.



- Learner ensembles are common approach to improve overall performance.
- Naive approach: Independently train K independent models using the same, standard discriminative loss \mathcal{L}_{disc}



- Learner ensembles are common approach to improve overall performance.
- Naive approach: Independently train K different models using the same, standard discriminative loss \mathcal{L}_{disc}
- Assumption: Aggregation over different embeddings increases model robustness.





- Learner ensembles are common approach to improve overall performance.
- Naive approach: Independently train K different models using the same, standard discriminative loss \mathcal{L}_{disc}
- Assumption: Aggregation over different embeddings increases model robustness.



- Learner ensembles are common approach to improve overall performance.
- Naive approach: Independently train K different models using the same, standard discriminative loss \mathcal{L}_{disc}
- Assumption: Aggregation over different embeddings increases model robustness.



BIER: Online Gradient Boosting [Opitz et al. 2018]: implicit specialization of learners





- Typically training distribution is **multimodal**
- **Explicit** data specialization of learners following **Divide & Conquer strategy**⁹



- Typically training distribution is **multimodal**
- **Explicit** data specialization of learners following **Divide & Conquer strategy**⁹



 $\mathcal{L}_{\text{disc}}$

⁹Sanakoyeu, Tschernetzski, Büchler, Ommer 2019

- Typically training distribution is **multimodal**
- Explicit data specialization of learners following Divide & Conquer strategy⁹



⁹Sanakoyeu, Tschernetzski, Büchler, Ommer 2019

Postprocessing: Joint global finetuning of concatenated learners to improve consistency of embeddings.



Postprocessing: Joint global finetuning of concatenated learners to improve consistency of embeddings.

⁹Sanakoyeu, Tschernetzski, Büchler, Ommer 2019

- Typically training distribution is **multimodal**
- **Explicit** data specialization of learners following **Hierarchical Divide & Conquer strategy**¹⁰



Sequential Splitting: Continuous specialization by splitting data and adding learners during optimization

¹⁰Sanakoyeu, Ma, Tschernetzski, Ommer 2021





Target similar object features!

• capture features **separating between classes** • aggregate features into 'classes' • e.g. "Ferrari" vs. "Hummer"

Assumption: Different features improve robustness to OOD data (new classes, etc.) \bullet



Class-discriminative features

• capture features separating between classes • aggregate features into 'classes' • e.g. "Ferrari" vs. "Hummer"

Learn about different/more general features?

- **Assumption**: Different features improve robustness to OOD data (new classes, etc.)
- Only class-labels available: use **unsupervised learning**



$\mathcal{L}_{ ext{disc}}$

Class-discriminative features

• capture features separating between classes • aggregate features into 'classes' • e.g. "Ferrari" vs. "Hummer"

- **Assumption**: Different features improve robustness to OOD data (new classes, etc.)
- Only class-labels available: use **unsupervised learning**



• capture features separating between classes • aggregate features into 'classes'

• anchors and positives from same cluster represent inter-class characteristics,

- **Assumption**: Different features improve robustness to OOD data (new classes, etc.)
- Only class-labels available: use **unsupervised learning**



¹²Milbich, Roth, Brattoli, Ommer 2020

$\mathcal{L}_{ ext{disc}}$

Class-discriminative features

• capture features separating between classes • aggregate features into 'classes' • e.g. "Ferrari" vs. "Hummer"

Class-shared features¹²

• anchors and positives **from different classes** • represent shared, general object

 $\mathcal{L}_{ ext{shared}}$

- Explicitlt sample anchor and positive from - Can be achieved by simply changing triplet sampling

- **Assumption**: Different features improve robustness to OOD data (new classes, etc.)
- Only class-labels available: use **unsupervised learning**





¹²Milbich, Roth, Brattoli, Ommer 2020

Class-discriminative features

• capture features separating between classes • aggregate features into 'classes' • e.g. "Ferrari" vs. "Hummer"

Class-shared features¹²

• anchors and positives from different classes • represent shared, general object

 $\begin{array}{c} t = \{I_a, I_p, I_n\} \ \hline y_a
eq y_p
eq y_n \end{array} \hspace{0.1in} \mathcal{L}_{ ext{shared}} \end{array}$

DiVA¹³: Diverse Visual Feature Aggregation: Multi-task ensemble with each learner focussing on complementary features





¹³Milbich, Roth, Bharadhwaj, Sinha, Y.Bengio, Ommer, Cohen 2021

DiVA¹³: Diverse Visual Feature Aggregation: Multi-task ensemble with each learner focussing on complementary features



¹³Milbich, Roth, Bharadhwaj, Sinha, Y.Bengio, Ommer, Cohen 2021



Class-shared features

 $y_a
eq y_p
eq y_n$

Multi-task ensemble with each learner focussing on complementary features



Multi-task ensemble with each learner focussing on complementary features



¹³Milbich, Roth, Bharadhwaj, Sinha, Y.Bengio, Ommer, Cohen 2021

Deep Metric Learning (DML)



Input space (Images)

Next talk:

Ranking-based DML

Classification-based DML

Proxy-based DML

Contextual DML

Ensemble-based DML

Others

• **S2SD**¹⁴: Learn ensemble of teachers and distill their 'knowledge' into student



¹⁴Roth, Milbich, Ommer, Cohen, Ghassemi 2021

• **S2SD**¹⁴: Learn ensemble of teachers and distill their 'knowledge' into student



- S2SD¹⁴: Learn ensemble of teachers and distill their 'knowledge' into student
- use different embedding dimensionalities for teachers to increase robustness of student



Knowledge distillation: enforce consistent similarity matrices

¹⁴Roth, Milbich, Ommer, Cohen, Ghassemi 2021

ent ess of student

Visual Similarity Learning

Given data distribution \mathcal{X}_{train} , infer pairwise semantic similarity.



Infer pairwise **similarity**

Input space (Images)
Visual Similarity Learning

Given data distribution \mathcal{X}_{train} , infer pairwise semantic similarity.

Infer pairwise similarity



Input space (Images)







Similarity?

similar	
dissimilar	Color
similar	
dissimilar	Viewpoint

Visual Similarity Learning

Given data distribution \mathcal{X}_{train} , infer pairwise semantic similarity.



Input space (Images)

Infer pairwise similarity

indicated by class labels







Similarity?

similar	
dissimilar	Class

Visual Similarity Learning

Learn **representation** $\phi(x)$ which reflects **semantic similarity** $d(\phi(x_i), \phi(x_j))$ within training distribution $\mathcal{X}_{\text{train}}$.









Input space (Images)

Similarity?

Ensemble-based DML

- Typically there are **different notions of similarity** (classes, color, viewpoint, ...)
- Assumption: Different features improve robustness to OOD data (new classes, etc.)
- **Explicit** similarity specialization of learners
- Only class-labels available: use **unsupervised learning**





• capture features separating between classes • aggregate features into 'classes'

• anchors and positives from same cluster represent inter-class characteristics,

Learn **representation** Φ which reflects semantic similarity within training distribution \mathcal{X}_{train} .





Input space (Images)



Learn **representation** $\phi(x)$ which reflects **semantic similarity** within training distribution $\mathcal{X}_{\text{train}}$.



Input space (Images)





Deep Metric Learning (DML)

Classification

Surrogate Tasks

Representation Learning

Learn **representation** Φ which reflects semantic similarity within training distribution \mathcal{X}_{train} .





Input space (Images)



Introduction DML



(Out-Of-Distribution-Generalization)

How well does Φ capture **unseen** classes, **unknown** surroundings, viewpoints, continual class **changes**?





Input space (Images)

(Out-Of-Distribution-Generalization)

How well does Φ capture **unseen** classes, **unknown** surroundings, viewpoints, continual class **changes**?



(Out-Of-Distribution-Generalization)

How well does Φ capture **unseen** classes, **unknown** surroundings, viewpoints, continual class **changes**?



(Out-Of-Distribution-Generalization)

How well does Φ capture **unseen** classes, **unknown** surroundings, viewpoints, continual class **changes**?

Evaluation needs to consider **broad range of test distributions** and **difficulty.**





Negative Sampling in Ranking-based DML



⁴ PADS: Policy-Adapted Sampling for Visual Similarity Learning, CVPR 2020

Negative Sampling in Ranking-based DML



⁴ PADS: Policy-Adapted Sampling for Visual Similarity Learning, CVPR 2020

Negative Sampling in Ranking-based DML



⁴ PADS: Policy-Adapted Sampling for Visual Similarity Learning, CVPR 2020

³ Roth, Milbich et al.; ICML 2021; S2SD: Simultaneous Similarity-based Self-distillation for Deep Metric Learning

Ensemble-based DML

• Multi Self-Distillation³



Ensemble-based DML

- Shared Features, DiVA, MIC (complementary semantics, "multi-task learning")
- **explicit** specialization of learners (semantics)



Overview: Typical Learning Concepts for DML Generalization

- Diverse Feature Aggregation²
- Multi Self-Distillation
- Proxy-Learning



² Milbich, Roth et al.; ECCV 2020; DiVA: Diverse Visual Feature Aggregation for Deep Metric Learning

